Constructing 2D Watermarks by Composition

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Abstract

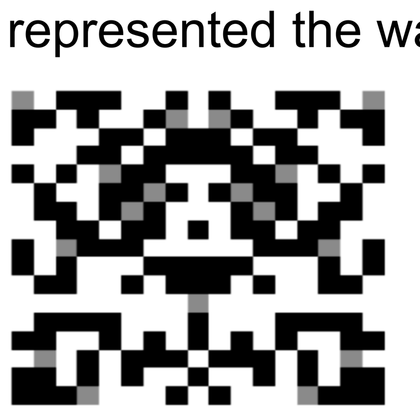
This study presented a method of constructing 2D periodic arrays by composing a 1D periodic array with a sequence of shifts calculated as a polynomial of order $n > 1$, ($\phi_1(x) \bmod p$), with coefficients from Z_p. The array construction is algebraic, based on finite fields, resulting in arrays with good correlation properties. This research illustrated such method using a Legendre sequence as a base array. The resulting 2D array had a peak auto-correlation value of $p(p-1)$ and a non-peak auto-correlation value of $-p$, 0. Finally, experimental results presented how to construct these watermarks and their resistance to Rotation (180-degrees), Salt, and Gaussian attacks.

Introduction

Sequences with good correlation have been studied and have many applications in modern communications, like radars, sonars, and digital watermarking (Tirkel and Hall [3]). This research considers arrays with good correlation properties and arrays constructed by composition where each column is a cyclic shift of one pseudo-noisy column or constant column. A sequence is considered pseudo-noisy if its autocorrelation takes on two values: a high value at zero shift and a single low value for all other shifts. The poster presents an implementation and analysis of watermarks constructed using shift sequences based on polynomials and Legendre sequence as a base array and shows some results of the robustness of this digital watermarking in images.

Methodology

Python and its variety of computational libraries were used to implement the watermarks. The first step consisted of writing code that generated Legendre sequence s base array. Given the size of the sequence p (a prime number), the function proceeds to create a finite field Z_p. Each value i in the s is obtained by the following criteria: 1, if i is quadratic residue modulo p. 0, if i is identical to 0 modulo p. -1, if i is not quadratic residue modulo p. To construct the watermark, the base array was shifted for all elements in Z_p. The number of shifts ($\phi_1(x) \bmod p$ places to the right) were determined by evaluating all x coefficients in Z_p in a polynomial $\phi_1(x)$ of order $n > 1$, whcih resulted in a p x p 2D matrix array that represented the watermark.



$p = 17$

$\phi_1(x) = 7x^2 + 7x$

Figure 1: 2D array obtained by using the following prime and polynomial

The watermarks were inserted by directly manipulating the pixels of the image and adding the pixel values of the watermark onto the image. By scaling the matrix with a positive integer k, the watermark's perceptibility increased. The position of watermark's insertion was dictated by random coordinates and all watermarks were inserted in the RGB image's green layer. The next part of the research consisted of attacking the watermarked images with Python class that modified the image's pixels. The attacks used were: Salt, Rotation (180 degrees), and Gaussian Filter. The following attacked watermarked images used k = 50:

Figure 2: 180 Attack

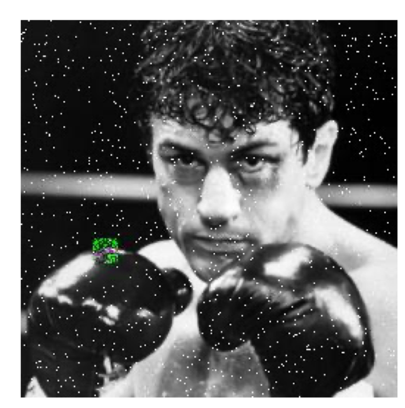


Figure 3: Salt Attack

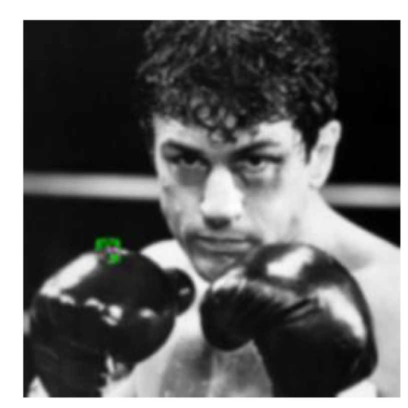


Figure 4: Gaussian Attack

2D matrix correlation is used to study how signals resemble each other (H.L. [1]). In essence, the watermark worked as a reference to find itself in an image. By displacing through all pixels of the watermarked image and cross-correlating it with the watermark, the result was a signal histogram with the correlation values obtained. This method is ideal to make sure that the watermarks are preserving their correlation properties.

Results

After inserting the watermarks and attacking the images, all of them maintained the essential properties and were able to be detected with the 2D correlation function. A series of simulations performed found that as the size of watermark and its k value increased, so did the visual perceptibility and peak values of its correlation. The minimum sized legendre watermark that maintained its properties was using a prime equal to 17 and a k value of 15. Upon simulating watermarks smaller than 17x17 and testing their correlation properties resulted in inconsistent signal histograms. The following images demonstrate such findings by using a correlation analysis on the 3 types of attacks used with 3 different k values: 25, 50, and 100.

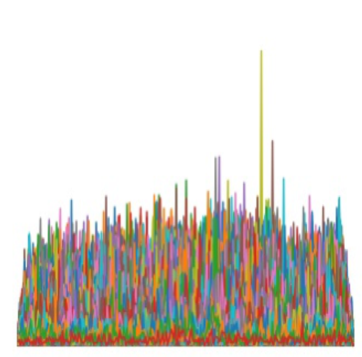


Figure 5: Gauss Correlation Histogram (k = 25)

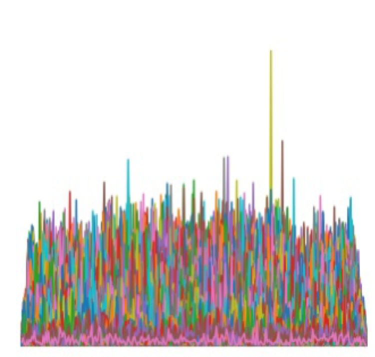


Figure 6: Rotate 180 Correlation Histogram (k = 25)

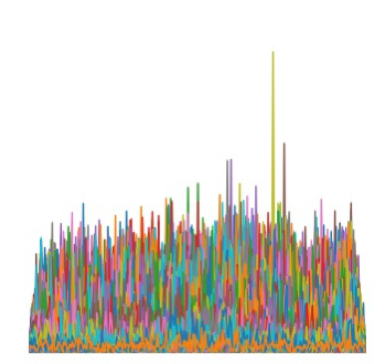


Figure 7: Salt Correlation Histogram (k = 25)

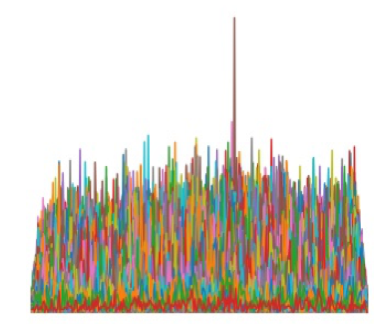


Figure 8: Gauss Correlation Histogram (k = 50)

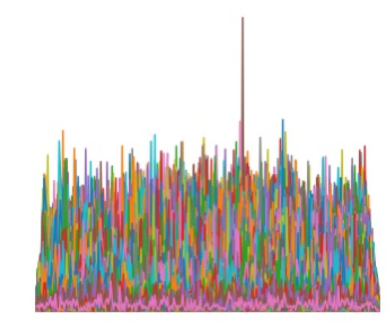


Figure 9: Rotate 180 Correlation Histogram (k = 50)

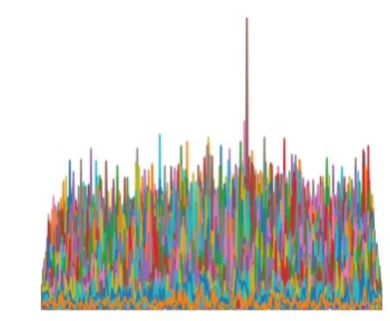


Figure 10: Salt Correlation Histogram (k = 50)

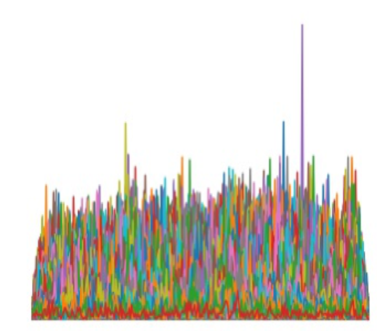


Figure 11: Gauss Correlation Histogram (k = 100)

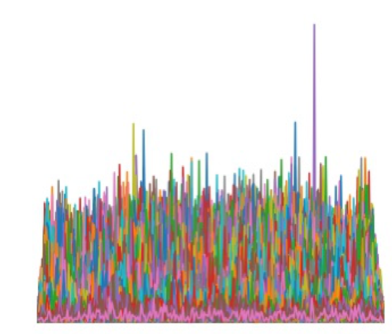


Figure 12: Rotate 180 Correlation Histogram (k = 100)

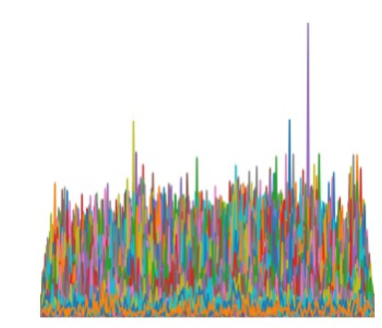


Figure 13: Salt Correlation Histogram (k = 100)

Conclusion and Future Work

The simulations performed showed that Legendre sequences are a good type of watermark that can resist a few types of attacks, given that the sequence has an adequate size and scalar value. It's important to clarify that the sequences generated in this research are resistant to a few attacks, but does not imply that it may resist all types of attacks. The next research step would be to expand the types of attacks on images and implement other sequences that may be a better fit to protect images against a higher number of attacks while minimizing its size and scalar value.

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References

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